A 200 gallon (gal) tank initially contains 30 pounds (lbs) of salt dissolved in 150 gallons of water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 4 gallons per minute, and the mixture, kept uniform by stirring, is pumped out at the same rate.

1. Set up an initial value problem to model this scenario.
2. Use the differential equation from (1) to decide whether the amount of salt in the tank is increasing or decreasing at time t=0 (you should not need to solve the diffeq to answer this!)
3. Now solve the IVP from (1) by using the "undetermined coefficients" method.

(1) $\frac{dx}{dt} = 8 - \frac{4x(t)}{150}$, $x(0) = 30$. "IVP"

(2) Since $x(0) = 30$, $\left.\frac{dx}{dt}\right|_{t=0} = 8 - \frac{4(30)}{150} > 0$, so x is increasing at time $t=0$.

(3) $x_h(t) = Ce^{-\frac{2t}{75}}$

$x_p(t) = A$
$x_p'(t) = 0$ $\Rightarrow$ $x' = 8 - \frac{2x}{75}$ becomes

$$0 = 8 - \frac{2A}{75}$$
$$A = 300$$

$\therefore$ $x_h(t) = Ce^{-2t/75}$ and $x_p = 300$, so

$$\boxed{x(t) = Ce^{-2t/75} + 300.}$$

Now use $x(0) = 30$:

$30 = Ce^0 + 300$

$\Rightarrow C = -270$

and $\boxed{x(t) = 300 - 270e^{-2t/75}.}$

A 200 gallon (gal) tank initially contains 30 pounds (lbs) of salt dissolved in 150 gallons of water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 4 gallons per minute. Meanwhile, a leaky pipe drips pure water into the tank at the rate of 1/2 gallon per minute. The mixture, kept uniform by stirring, is now being pumped out of the tank at the rate of 4.5 gallons per minute.

1. Set up an initial value problem to model this scenario.
2. Now solve the IVP from (1) by using the "integrating factor" method.
3. At what time does the amount of salt in the tank reach 100 pounds?

(1) $\frac{dx}{dt} = 8 - \frac{3x}{100}, \quad x(0) = 30$

(2) $\frac{dx}{dt} + \frac{3x}{100} = 8$

Integrating factor: $e^{\int \frac{3}{100}\,dt} = e^{\frac{3t}{100}}$.

$$\underbrace{\frac{dx}{dt} e^{\frac{3t}{100}} + \frac{3x}{100} e^{3t/100}} = 8e^{3t/100}$$

If you've chosen int. factor correctly, this will be

$$\frac{d}{dt}\left(x(t)\cdot(\text{int factor})\right) = 8e^{3t/100}.$$

$$\int \frac{d}{dt}\left(x\cdot e^{3t/100}\right)dt = \int 8e^{3t/100}\,dt$$

$$x\cdot e^{3t/100} = \frac{800}{3}e^{3t/100} \boxed{+C}$$

this must show up here.

$$x(t) = \frac{800}{3} + Ce^{-3t/100}$$

$x(0) = 30$

$$30 = \frac{800}{3} + Ce^{0} \Rightarrow C = -\frac{710}{3}.$$

$$\boxed{x(t) = \frac{800}{3} - \frac{710}{3}e^{-3t/100}.}$$

A 200 gallon (gal) tank initially contains 30 pounds (lbs) of salt dissolved in 150 gallons of water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 4 gallons per minute. The mixture, kept uniform by stirring, is now pumped out of the tank at the rate of 4 gallons per minute, but unbeknownst to the quality control department, also leaks out at a rate of 2 gallons per minute.

1) Note that the tank is draining faster than it is being filled. At what time will it be empty? Since the amount of salt starts at 30 pounds, is continuously changing, and ends at 0 pounds, there must be some time at which the amount of salt in the tank reaches a maximum. Let's solve the equation to find this maximum (move on to part (2)).

2) Set up an initial value problem to model the scenario.

3) Solve the IVP from (2) by using the "integrating factor" method.

4) Use your calculator to plot the solution on a reasonable time interval (based on your answer to #2), and determine the maximum amount of salt in the tank and the time at which the maximum is achieved.

$$\frac{dx}{dt} + \frac{6x}{150-2t} = 8$$

int. factor

$$\rho(t) = e^{\int \frac{6}{150-2t}dt}$$

$$= e^{-\frac{6}{2}\ln|150-2t|}$$

$$e^{a\cdot b} = (e^a)^b$$

$$= e^{-3\ln|150-2t|}$$

Since tank is empty at $t=75$,

$$\rho(t) = (150-2t)^{-3}$$

$$\underbrace{\frac{dx}{dt}\cdot\frac{1}{(150-2t)^3} + \frac{6x}{150-2t}\cdot\frac{1}{(150-2t)^3}} = \frac{8}{(150-2t)^3}$$

$$x(t)\cdot\frac{1}{(150-2t)^3} = -4\int\frac{-2}{(150-2t)^3}dt$$

$u = 150-2t$

$du = -2dt$

$$-4\int u^{-3}du = \frac{-4u^{-2}}{-2} + C$$

$$\frac{2}{(150-2t)^2} + C$$

$$x(t)\cdot\frac{1}{(150-2t)^3} = \frac{2}{(150-2t)^2} + C$$

$$x(t) = 2(150-2t) + C(150-2t)^3$$

$$30 = 2(150-0) + C(150-0)^3$$

$$\Rightarrow \frac{30-300}{(150)^3} = C \qquad \frac{-270}{(150)^3}$$

$$x(t) = 2(150-2t) - \frac{270}{(150)^3}(150-2t)^3$$

A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of

(1/5)(1+cos(t)) lbs/gal. If a well mixed solution leaves the tank at a rate of 6 gal/hr, how much salt is in the tank when it overflows?