$$\frac{dy}{dt} = -3y + 4e^{2t} - \sin t \qquad y(0) = 1$$

① Find y_h. (sol. to associated homogeneous eqn.)

$$\frac{dy}{dt} = -3y$$

$$\boxed{y_h = Ce^{-3t}}$$

If $\frac{dy}{dt} = k \cdot y$, $y = Ce^{kt}$

If $\frac{dy}{dt} = a(t) \cdot y$, $y = Ce^{\int a(t)dt}$

(separate if you need to or if you want to be sure)

$$\int \frac{dy}{y} = \int -3\,dt$$

$$\ln|y| = -3t + C$$

$$|y| = e^C e^{-3t}$$

$$|y| = \pm e^C e^{-3t}$$

$$y = ke^{-3t}$$

② now find y_p. "particular solution"

$$\frac{dy}{dt} = -3y + \underbrace{4e^{2t} - \sin t}$$

guess y_p: $y_p = Ae^{2t} + B\sin t + C\cos t$

$$\frac{dy}{dt} + 3y = 4e^{2t} - \sin t$$

$$\underbrace{\left(2Ae^{2t} + B\cos t - C\sin t\right)} + 3\underbrace{\left(Ae^{2t} + B\sin t + C\cos t\right)} = 4e^{2t} - \sin t$$

Equate coefficients:

e^{2t} : $5A = 4$ $\qquad A = 4/5$

$\cos t$: $B + 3C = 0$ $\qquad B = -3/10$

$\sin t$: $-C + 3B = -1$ $\qquad C = 1/10$

$$\begin{cases} B + 3C = 0 \\ 9B - 3C = -3 \end{cases}$$

$10B = -3$

$B = -3/10$

$C = 1/10$

So $y_p = \frac{4}{5}e^{2t} - \frac{3}{10}\sin t + \frac{1}{10}\cos t$

③ "The" general solution to the original eqn (nonhomogeneous) is $y = y_p + y_h$

$$= \underbrace{\frac{4}{5}e^{2t} - \frac{3}{10}\sin t + \frac{1}{10}\cos t}_{y_p} + \underbrace{Ce^{-3t}}_{y_h}$$

Assign 5 #4:

$$\frac{dy}{dt} + y = t^3 + \sin 3t$$

$$\boxed{y_h = Ce^{-t}}$$

$$y_p = \underbrace{At^3 + Bt^2 + Ct + D}_{\text{b/c of } t^3} + \underbrace{E\sin 3t + F\cos 3t}_{\text{b/c of } \sin 3t}$$