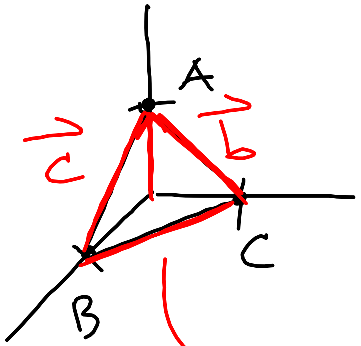


Prove $\triangle ABC$ is equilateral.

$$\vec{a} = \vec{B} - \vec{C} = \begin{pmatrix}0\\1\\0\end{pmatrix} - \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}-1\\1\\0\end{pmatrix}$$

$$\vec{b} = \vec{A} - \vec{C} = \begin{pmatrix}0\\0\\1\end{pmatrix} - \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$\vec{c} = \vec{A} - \vec{B} = \begin{pmatrix}0\\0\\1\end{pmatrix} - \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\-1\\1\end{pmatrix}$$

(26) $|\vec{u}\cdot\vec{v}| = |\vec{u}||\vec{v}|$ if and only if $\vec{u}$ and $\vec{v}$ are lin. dependent.

Assume $\vec{u}$ & $\vec{v}$ are linearly dependent.

Then $\vec{u} = k\vec{v}$ for some k.

$$|k\vec{v}\cdot\vec{v}| \stackrel{?}{=} |k\vec{v}||\vec{v}|$$

$= |k|\,|\vec{v}\cdot\vec{v}|$ → used $\vec{v}\cdot\vec{v} = |\vec{v}|^2$.

$= |k|\,|\vec{v}|^2$ $\qquad = |k||\vec{v}||\vec{v}| = |k||\vec{v}|^2$.

Now assume $|\vec{u}\cdot\vec{v}| = |\vec{u}||\vec{v}|$.

$$\frac{|\vec{u}\cdot\vec{v}|}{|u||\vec{v}|} = 1$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \pm 1$$

$\Rightarrow \theta = 0°$ or $180°$, in which case the vectors are lin. dep.

Show that the only vector in V & $V^{\perp}$ is the zero vector.

Let $\vec{v} \in V$, and $\vec{v} \in V^{\perp}$.

$$\vec{v} \cdot \vec{v} = 0.$$

$$|\vec{v}|^2 = 0 \Rightarrow \vec{v} = \vec{0}.$$

Prove $\vec{0}$ must be in every subspace:

WRONG >> ~~$k\underline{\vec{0}} = \vec{0}$. $\therefore$ $\vec{0}$ is in the space.~~

CORRECT >>

- Let $\vec{v} \in V$. Then $k\vec{v} \in V$ for all $k \in \mathbb{R}$,

$\therefore$ $0\vec{v} = \vec{v} \in V$.

$$T(x, y) = (3x + y,\ 2y,\ x + y)$$

$\mathbb{R}^2$ $\qquad$ $\mathbb{R}^3$

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x + y \\ 2y \\ x + y \end{bmatrix}$$

$$A \quad \vec{x} = T(x, y)$$

The range of (all possible "images") of A linear transformation just describes the column space of the transformation matrix

$$T(\square, \triangle) = (3\square+\triangle, 2\triangle, \square+\triangle)$$

T is linear if

$$\left.\begin{array}{l} T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v}) \\ \text{and} \\ T(k\vec{v}) = k \cdot T(\vec{v}) \end{array}\right\}$$

Let
$\vec{u} = (a, b)$
$\vec{v} = (x, y)$

$$T(\vec{u}+\vec{v}) = T(a+x, b+y)$$

$$= (3a+3x+b+y, 2b+2y, a+x+b+y)$$

$$T(\vec{u}) + T(\vec{v}) = \underbrace{(3a+b, 2b, a+b)}_{T(\vec{u})} + \underbrace{(3x+y, 2y, x+y)}_{T(\vec{v})}$$

$$= (3a+b+3x+y, 2b+2y, a+x+b+y)$$

$\therefore T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ ✓

Now, $T(k\vec{v}) = T(kx, ky) = (3kx+ky, 2ky, kx+ky)$

$$kT(\vec{v}) = kT(x,y) = k(3x+y, 2y, x+y)$$

$\therefore T(k\vec{v}) = kT(\vec{v})$.

$T(x,y)$ is linear!

Is $T(x, y) = (x^2, y)$ linear?

Does $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$?

$T(\vec{u}+\vec{v}) = T(a+x, b+y) = ((a+x)^2, b+y)$

$\hookrightarrow a^2+2ax+x^2$

$T(\vec{u}) + T(\vec{v}) = T(a, b) + T(x, y)$

$= (a^2, b) + (x^2, y) = (a^2+x^2, b+y)$

$\therefore$ $T(x, y) = (x^2, y)$ is <u>not</u> linear.

$T_1(x, y) = (2x + 3y,\ x - y,\ x + 2y)$

$T_2(x, y, z) = (x - y + z)$ $\qquad R^2 \to R^3 \to R^1$

$$T_2 \circ T_1\ (x) = T_2(T_1(x)).$$

$\vec{u} = (3, 4)$

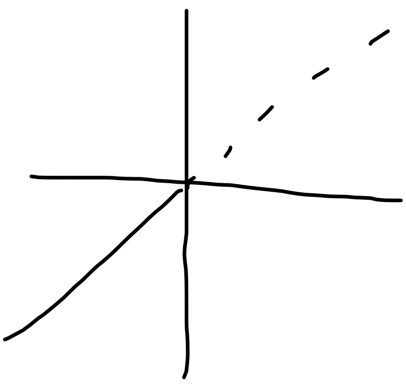
$(T_2 \circ T_1)(\vec{u})$

$$\begin{array}{ccc} A_2 & A_1 & \vec{u} \\ \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} & \begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 1 & 2 \end{bmatrix} & \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \end{array}$$

$$= \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 18 \\ -1 \\ 11 \end{bmatrix}$$

$$= \begin{bmatrix} 30. \end{bmatrix}$$

$y = z$



(x, y, z)

$0x + y - z = 0$

$(1,0,0)$, $(0,1,1)$

Set of all solutions to this eqn = subspace spanned by this plane.

(x, y, y)

$(1, 0, 0)$

$(0, 1, 1)$

$0x + y - z = 0$

Let $x = t$, $z = u$.

$y = u$

$(t, u, u) = t(1,0,0) + u(0,1,1)$

Let $\mathbf{x}$ be an arbitrary vector in $\mathbf{R}^n$ and $\mathbf{y}$ $\boxed{\text{a fixed vector.}}$ Prove that the transformation $T(\mathbf{x}) = \mathbf{x} \cdot \mathbf{y}$, (i.e., the *dot product* of $\mathbf{x}$ and $\mathbf{y}$) is a linear transformation of $\mathbf{R}^n \to \mathbf{R}$.

Is $T(\vec{x}+\vec{z}) = T(\vec{x}) + T(\vec{z})$?

$$\left\{\begin{array}{l} T(\vec{x}) = \vec{x}\cdot\vec{y}. \\ T(\vec{z}) = \vec{z}\cdot\vec{y}. \\ T(\vec{x})+T(\vec{z}) = \vec{x}\cdot\vec{y} + \vec{z}\cdot\vec{y} \\ T(\vec{x}+\vec{z}) = (\vec{x}+\vec{z})\cdot\vec{y} = \vec{x}\cdot\vec{y}+\vec{z}\cdot\vec{y}. \end{array}\right.$$

Is $T(k\vec{x}) = kT(\vec{x})$?

V_i are lin. ind.

$\vec{u}_1 = \vec{v}_1 + \vec{v}_2$, $\vec{u}_2 = 2\vec{v}_1 + 3\vec{v}_2$.

Does $a\vec{u}_1 + b\vec{u}_2 = \vec{0} \implies a, b = 0$?

Find a basis for the orthogonal complement to the vectors
$\{(0,1,2,3,3), (2,1,1,1,2)\}$.

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 3 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 & 3 \end{bmatrix}$$

$$\longrightarrow \begin{array}{c} \begin{bmatrix} 2 & 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 3 & 3 \end{bmatrix} \\ \begin{array}{ccccc} & & t & u & v \end{array} \end{array}$$

$2x_1 = t + 2u + v$

$x_1 = \frac{t}{2} + u + \frac{v}{2}$

$x_2 = -2t - 3u - 3v$

x_1	x_2	t	u	v
1	-4	2	0	0
1	-3	0	1	0
1	-6	0	0	2