Quick recap part 12:

Fundamental Matrix Subspaces:

Basis for the row space of A - take non-zero rows of RREF(A).

Basis for the col space of A - take the columns of A corresponding to the "pivot columns" of RREF(A). MUST use original columns.

Basis for the null space of A - find a basis for the solution space of the corresponding system $A\mathbf{x}=\mathbf{0}$.

Basis for the left null space of A - find a basis for the solution space of the corresponding system $A^T\mathbf{x}=\mathbf{0}$.

Quick recap part 12:

Using Fundamental Matrix Subspaces:

Let S = {v1, v2, v3, ..., vk}

Find a basis for the subspace spanned by S.

(Basis spans S and is linearly independent.)
The Span of S consists of all linear combinations of the vi in S. We would set the vi as columns and find a basis for the column space, or we could set the vi as rows and find a basis for the row space.

Quick recap part 12:

Using Fundamental Matrix Subspaces:

Let S = {v1, v2, v3, ..., vk}

Find some subset of S that forms a basis for the subspace spanned by S.

We would set the vi as columns and find a basis for the column space.

Quick recap part 12:

Using Fundamental Matrix Subspaces:

Let S = {v1, v2, v3, ..., vk}, where k<n.

Extend the set S to form a basis for $\mathbf{R}^n$.

Example: S = {(1,2,3,4), (2,1,1,0))}. Find a basis for R^4.

We would set the vectors in S as columns, then "append" the 4x4 identity matrix and rref as usual - the rref process will identify which 4 vectors are independent.

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RREF(A) = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & \cdots \\ 0 & 1 & 0 & 0 & \cdots & \cdots \\ 0 & 0 & 1 & 0 & \cdots & \cdots \\ 0 & 0 & 0 & 1 & \cdots & \cdots \end{bmatrix}$$

Basis: {(1,2,3,4), (2,1,1,0), (1,0,0,0), (0,1,0,0)}.

Find a basis for the subspace of R^3 spanned by the plane $x + 2y - 3z = 0$.

Solve this system. (solve $A\vec{x} = \vec{0}$)

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \end{array}\right]$$

$x = -2t + 3u$

$y = t$

$z = u$

$$= t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}.$$

3. Find a basis for the span of the following vectors.

   (a) $S = \{(1,1,1), (2,1,0), (0,1,1), (1,2,2)\}$
   (b) $S = \{(3,1,-1,0), (0,-1,2,-1), (4,3,8,3)\}$
   (c) $S = \{(1,1,2), (1,2,1), (-1,-7,4)\}$

4. Find a basis for the orthogonal complement to each of the subspaces spanned by the vectors in Problems 3a through 3c.

null(A) $\perp$ row(A).

Let $A = \begin{bmatrix} 3 & 1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 4 & 3 & 8 & 3 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Prove that $T(x,y) = (x-y, 3x)$ is linear.

Does $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$?

Let $\vec{u} = (a, b)$, $\vec{v} = (c, d)$

$T(\vec{u}) = (a-b, 3a)$

$T(\vec{v}) = (c-d, 3c)$

$$\boxed{\begin{aligned} T(\vec{u}) + T(\vec{v}) &= (a-b, 3a) + (c-d, 3c) \\ &= (a+c-b-d, 3a+3c) \end{aligned}}$$

Meanwhile...

$\vec{u}+\vec{v} = (a+c, b+d)$

$$\boxed{T(\vec{u}+\vec{v}) = (a+c-(b+d), 3(a+c))}$$

The results are equivalent.

② Is $T(k\vec{u}) = kT(\vec{u})$?

$T(k\vec{u}) = T(ka, kb) = (ka-kb, 3ka)$

and

$kT(\vec{u}) = k(a-b, 3a) = (k(a-b), 3ka)$

equivalent.

$\therefore T(x,y) = (x-y, 3x)$ is a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$.

Prove that the transformation $T(x, y, z) = (xy, z)$ *from* R^3 *to* R^2 *is not linear.*

Let $\vec{u} = (a, b, c)$; $\vec{v} = (d, e, f)$

$T(\vec{u}) = (ab, c)$ $\quad T(\vec{v}) = (de, f)$.

$\vec{u} + \vec{v} = (a + d, b + e, c + f)$

$T(\vec{u} + \vec{v}) = ((a+d)(b+e), c+f)$

$= (ab + ae + db + de, c + f)$

Whereas $T(\vec{u}) + T(\vec{v}) = (ab + de, c + f)$.