- A. $\{(x, x-5, x+6) \mid x \text{ arbitrary number}\}$
- B. $\{(x,y,z) \mid x,y,z > 0\}$
- C. $\{(x,y,z) \mid x+y+z=-3\}$
- D. $\{(3x, -2x, 8x) \mid x \text{ arbitrary number}\}$
- E. $\{(x,y,z) \mid 5x-6y=0, -9x+3z=0\}$
- F. $\{(x,y,z) \mid x+y+z=0\}$

Let $\vec{t}, \vec{v} \in A$, and k a scalar.

then $\vec{t} = (t, t-5, t+6)$

$\vec{v} = (v, v-5, v+6)$

$\vec{t} + \vec{v} = (t+v, t+v-10, t+v+12)$

but this should be $t+v-5$.

(not closed under addition, not a subspace.)

- A. $\{(x, x-5, x+6) \mid x \text{ arbitrary number }\}$
- B. $\{(x, y, z) \mid x, y, z > 0\}$
- C. $\{(x, y, z) \mid x+y+z=-3\}$
- D. $\{(3x, -2x, 8x) \mid x \text{ arbitrary number }\}$
- E. $\{(x, y, z) \mid 5x-6y=0, -9x+3z=0\}$
- F. $\{(x, y, z) \mid x+y+z=0\}$

If $k=-1$, then $k(x,y,z)=(-x,-y,-z)$, and the components are not all positive.

- A. $\{(x, x-5, x+6) \mid x \text{ arbitrary number }\}$
- B. $\{(x,y,z) \mid x,y,z > 0\}$
- C. $\{(x,y,z) \mid x+y+z=-3\}$
- D. $\{(3x, -2x, 8x) \mid x \text{ arbitrary number }\}$
- E. $\{(x,y,z) \mid 5x-6y=0, -9x+3z=0\}$
- F. $\{(x,y,z) \mid x+y+z=0\}$

Let $\vec{u}=(a,b,c)$ and $\vec{v}=(d,e,f) \in C$, and $k \in \mathbb{R}$.

$$\vec{u}+\vec{v}=(\underbrace{a+d}_{\text{"}x\text{"}}, \underbrace{b+e}_{\text{"}y\text{"}}, \underbrace{c+f}_{\text{"}z\text{"}})$$

Is $(a+d)+(b+e)+(c+f)=-3$?

$$=(a+b+c)+(d+e+f)$$

$$=(-3)+(-3)$$

$$=-6 \neq -3.$$ (so <u>not</u> a subspace.)

- A. $\{(x, x-5, x+6) \mid x \text{ arbitrary number}\}$
- B. $\{(x,y,z) \mid x,y,z > 0\}$
- C. $\{(x,y,z) \mid x+y+z = -3\}$
- D. $\{(3x, -2x, 8x) \mid x \text{ arbitrary number}\}$
- E. $\{(x,y,z) \mid 5x-6y=0, -9x+3z=0\}$
- F. $\{(x,y,z) \mid x+y+z=0\}$

METHOD 1

$\vec{a} = (x,y,z)$ $\vec{b} = (t,u,v) \in E$

$\vec{a}+\vec{b} = (x+t, y+u, z+v)$

- Is $5(x+t)-6(y+u)=0$?

$(5x-6y)+(5t-6u) = 0+0=0.$ ✓

- Is $-9(x+t)+3(z+v)=0$?

$(-9x+3z)+(-9t+3v) = 0+0=0.$ ✓

METHOD 2

Represent vectors as having this structure.

$\vec{u} = (x, \frac{5}{6}x, 3x)$

$\vec{v} = (t, \frac{5}{6}t, 3t)$ $\in E.$

$\vec{u}+\vec{v} = \left(x+t, \frac{5}{6}(x+t), 3(x+t)\right)$

And $5(x+t) - 6\left(\frac{5}{6}(x+t)\right) = 0$

and $9(x+t) - 3(3(x+t)) = 0.$

$k\vec{u} = \left(k(x+t), \frac{5k}{6}(x+t), 3k(x+t)\right)$

$$\vec{u} = \begin{bmatrix} 3 \\ 7 \\ 16 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 1 \\ -4+k \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 3 \\ 1 \\ 10 \end{bmatrix}$$

are linearly independent if and only if $k \neq$ _____ .

Lin. ind. if $a\vec{u} + b\vec{v} + c\vec{w} = \vec{0}$

has only the trivial soln.

$$a\begin{bmatrix} \\ \\ \end{bmatrix} + b\begin{bmatrix} \\ \\ \end{bmatrix} + c\begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 7 & 1 & 1 \\ 16 & -4+k & 10 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- only trivial soln?
- is A inv.?
- is $|A| \neq 0$?
- is A row equiv. to I?

3.6.54 An "if and only if" proof requires both directions to be proven.

$\Rightarrow$: If both $\mathbf{A}$ and $\mathbf{B}$ are invertible, we know that $|\mathbf{A}| \neq 0$ and $|\mathbf{B}| \neq 0$. Then $|\mathbf{AB}| = |\mathbf{A}|\,|\mathbf{B}|$ cannot be zero (since neither factor is zero), so $\mathbf{AB}$ must also be invertible.

$\Leftarrow$: If $\mathbf{AB}$ is invertible, then $|\mathbf{AB}| \neq 0$. Since $|\mathbf{AB}| = |\mathbf{A}|\,|\mathbf{B}|$, we must have $|\mathbf{A}| \neq 0$ and $|\mathbf{B}| \neq 0$. Therefore, both $\mathbf{A}$ and $\mathbf{B}$ are invertible.

Thm 2: $n \times n$ matrix A is inv. iff $|A| \neq 0$.

Thm 3: $|AB| = |A||B|$.

3.6.55 Suppose $\mathbf{AB} = \mathbf{I}$. Then, from 3.6.54 we know that A and B are both invertible. If so, it must be true that $\mathbf{A}^{-1}\mathbf{AB} = \mathbf{A}^{-1}\mathbf{I}$, in which case, $\mathbf{IB} = \mathbf{A}^{-1}\mathbf{I}$, i.e., $\mathbf{B} = \mathbf{A}^{-1}$. (This is an important result - it shows that if we find an inverse for a square matrix from one side, we know it's the inverse from the other side as well.)

$\det(AB) = |A||B| = \det(I) = 1$.

<u>3.4.22</u>:

$$\begin{cases} x_1 - x_2 \quad\quad + 7x_4 + 3x_5 = 0 \\ \quad\quad\quad x_3 - x_4 - 2x_5 = 0 \end{cases}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & 7 & 3 & 0 \\ 0 & 0 & 1 & -1 & -2 & 0 \end{array}\right]$$

t x_2, u x_4, v x_5 $\longleftarrow$ free

x_1 x_3 $\longleftarrow$ leading.

$$x_3 = u + 2v$$

$$x_1 = t - 7u - 3v$$

"scalar form"

$x_1 = t - 7u - 3v$, $x_2 = t$, $x_3 = u + 2v$, $x_4 = u$, $x_5 = v$

"vector form"

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} t - 7u - 3v \\ t \\ u + 2v \\ u \\ v \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} -7 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + v \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Soln space of this system (subspace of $\mathbb{R}^5$) is the set of all linear combinations of

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$